A new approach to test Lorentz invariance

H.W. Wilschut, E.A. Dijck, S. Hoekstra, K. Jungmann,
S.E. Müller, J.P. Noordmans, C.J.G. Onderwater, C. Pijpker,
A. Sytema, R.G.E. Timmermans, K.K. Vos, and L. Willmann

Kernfysisch Versneller Instituut, University of Groningen,
Zernikelaan 25, 9747 AA Groningen, The Netherlands[*]

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Abstract

Lorentz invariance in the weak interaction has been tested rather poorly compared to the electromagnetic interaction. In this work we show which tests on the weak interaction should be considered. We focus on one particular test that explores the spin degree of freedom in β decay. To connect various phenomenological tests of Lorentz invariance in the weak interaction, we exploit a new theoretical model that may provide a framework that relates the different tests.

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[*]Electronic address: Wilschut@kvi.nl

There are many experimental tests of Lorentz invariance [1]. Arguably, the most precise tests have been done for the electromagnetic interaction. For the weak interaction, in particular for β decay, very few studies have been made. This despite the fact that the Standard Model (SM) originated from –and has been shaped by– the details of the weak interaction, i.e. the violation of Parity (P) and Charge conjugation (C) on one hand and the violation of the combined CP symmetry on the other.

Presently, one of the main efforts in fundamental physics is the unification of the Standard Model with General Relativity, in what is mostly referred to as quantum gravity models. Certain models of quantum gravity contain terms which violate Lorentz invariance and CPT symmetry (e.g. [2–6]). Manifestations of Lorentz Invariance Violation (LIV) could be searched for in low-energy experiments, such as in β decay. Requiring a theory that identifies the appropriate observables. Kostelecký and coworkers have developed a theoretical framework named "Standard-Model Extension" (SME) that contains all the properties of the Standard Model and General Relativity, but additionally contains all possible terms violating Lorentz and CPT symmetry via spontaneous breaking of Lorentz invariance [7]. It follows from this phenomenological approach, that observables for the different interactions are a priori independent. In this respect it is insufficient to test only the electromagnetic interaction.

We have started an experimental and theoretical program on LIV considering charged currents in the weak interaction, focusing on β decay. Recently a theoretical framework has been formulated that gives guidance to possible experiments [8]. It also shows to what extent various experiments could be related. In this theory the Lorentz symmetry breaking is implemented by modifying the propagation of the W boson. The theoretical motivation for this can be found in reference [8]. Here we will discuss the relevant results for β-decay experiments. In our experimental work we focus on the spin degree of freedom which up to now was not considered at all.

In the SM the β-decay rate, ignoring Coulomb and induced recoil effects, is given by [9]

$$\frac{d\Gamma}{\Gamma_0} = 1 + \vec{\beta} \cdot [A\frac{\langle \vec{J} \rangle}{J} + G\vec{\sigma}], \tag{1}$$

where $\vec{\beta}$ is the velocity of the β particle in units of the light velocity. $\frac{\langle \vec{J} \rangle}{J}$ is the degree of nuclear polarization of the parent nucleus and its direction. $\vec{\sigma}$ is the spin vector of the β particle. $\hbar/\Gamma_0$ is the lifetime of the nucleus. A and G are the well known parity violating

parameters: A is the β asymmetry or "Wu" parameter and G the longitudinal polarization of the outgoing β particle ($G = \pm 1$ for $\beta^{\mp}$).

If there is a preferred direction in space (i.e. Lorentz symmetry breaking), eq. (1) will be modified. In that case we expect it to be of the form

$$\frac{d\Gamma}{\Gamma_0} = 1 + \vec{\beta} \cdot [A \frac{\langle \vec{J} \rangle}{J} + \xi_1 \hat{n}_1 + G\vec{\sigma}] + \xi_2 \frac{\langle \vec{J} \rangle}{J} \cdot \hat{n}_2 + \xi_3 \vec{\sigma} \cdot \hat{n}_3. \tag{2}$$

Here $\hat{n}_i$ are the preferred directions in space. The directions do not need to be the same for the various observables, as we will show below. The ξ_i are the magnitudes of the Lorentz symmetry breaking terms. Because a measurement of $\vec{\sigma}$ inevitably involves measuring $\vec{\beta}$, we only consider ξ_1 and ξ_2. Where ξ_1 measures the degree of β-emission anisotropy of non-oriented nuclei, while ξ_2 measures the dependence of the lifetime on orientation. With respect to eq. 2 two important observations should be made. First, because of the low velocities of the parent nucleus we do not consider modifications of the decay rate due to boosts i.e. a dependence on absolute velocity. Second, the dependencies predicted in [8] go beyond those given in this equation, however, for the experiments we discuss here it suffices.

Concerning the β asymmetry, ξ_1, two measurements were made in the seventies [10, 11]. Both were made for forbidden decays. The main idea behind these experiments was to test rotational invariance by trying to observe violation of angular momentum conservation. A forbidden decay would become less forbidden to the extent that angular momentum would not be conserved, thus relatively enhancing the violating signal. By measuring the decay rate in various directions and correlating it with the earth's rotation, deviations from isotropy were searched for. To reach high precision, the whole setup including source and detector needed to be rotated. No deviations were found with a dependence of $\cos(\omega t)$ up to a level of 1.6×10^{-7} for the unique first forbidden transition in ^{90}Y [10], where ω is the earth's rotation frequency. The second experiment considered a second forbidden transition in ^{99}Tc [11] and reached a limit of 3×10^{-5}. It remains to be seen what these values mean in an underlying theory. It is presently evaluated [12] within the context of our theoretical work [8]. Some preliminary conclusions will be discussed below.

In our experimental work we are considering a polarization-dependent lifetime, parameterized by $\xi_2 \hat{n}_2$, for which no experimental information is available as yet [14]. It requires a sample of radioactive nuclei with oriented spin, for which the lifetime must be measured. We have searched for methods where these two tasks could be efficiently done. We found a

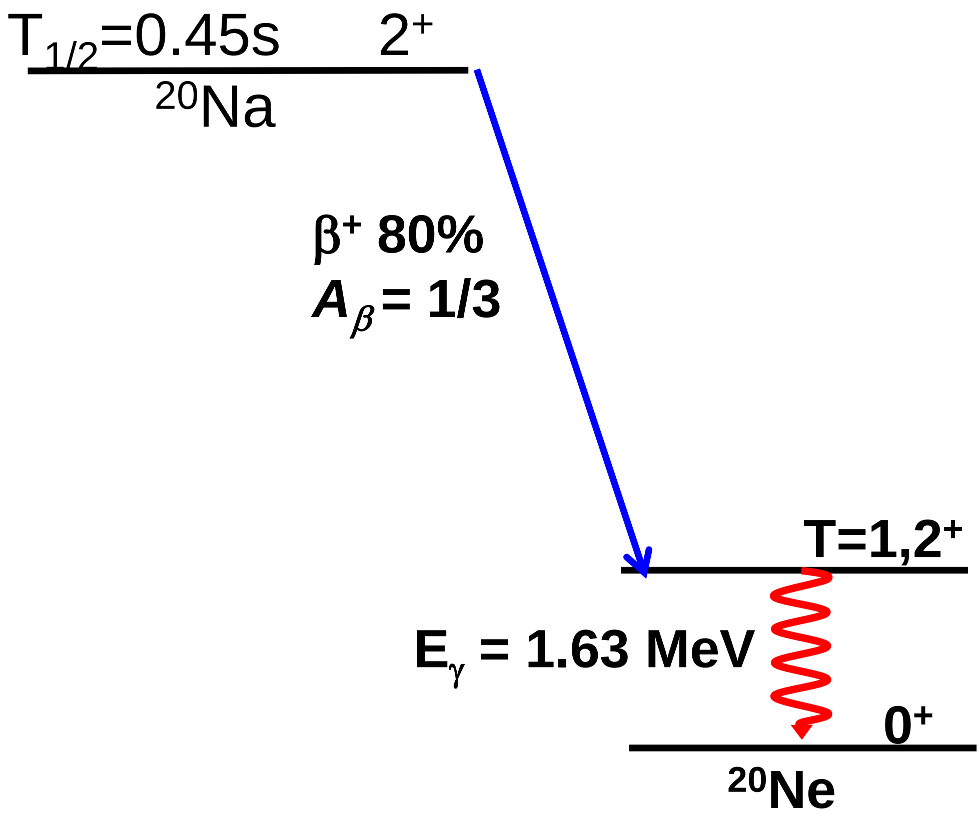

FIG. 1: Relevant part of the decay scheme of ^{20}Na

class of nuclei that allows one to measure the polarization independently from the lifetime. The nuclear polarization is best measured from the asymmetry parameter A. The lifetime can be obtained from measuring the depopulation of a nuclear excited state that is fed by the β decay. We show in fig. 1 schematically the situation for ^{20}Na as an example. We define $P\vec{j} = \frac{\langle \vec{J} \rangle}{J}$, where $\vec{j}$ is a unit vector in the direction of $\vec{J}$ and P the degree of polarization. Building an asymmetry by measuring the difference of decay rates R_β between the emission of β particles parallel and antiparallel to $\vec{J}$ with an analyzing power K, one determines P by

$$P = \frac{1}{AK} \frac{R_\beta^\uparrow - R_\beta^\downarrow}{R_\beta^\uparrow + R_\beta^\downarrow}, \tag{3}$$

which allows one to extract ξ_2 by measuring the γ-decay rates for the two polarization directions

$$\xi_2(\vec{j} \cdot \hat{n}_2) = \frac{1}{P} \frac{R_\gamma^\uparrow - R_\gamma^\downarrow}{R_\gamma^\uparrow + R_\gamma^\downarrow} = \frac{1}{P} \frac{\tau^\downarrow - \tau^\uparrow}{\tau^\uparrow + \tau^\downarrow}. \tag{4}$$

To be independent of possible changes in the sample sizes for the two spin directions, one

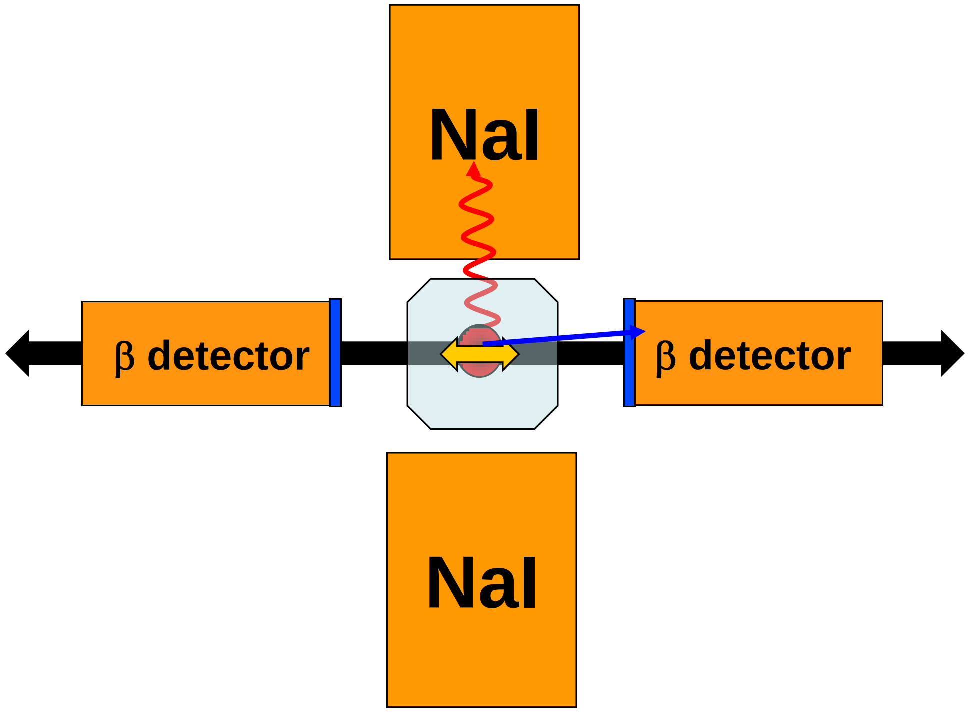

FIG. 2: Generic setup to measure rotational invariance violation in spin-polarized nuclei. The yellow arrow indicates the polarization directions of the radioactive sample. The complete setup needs to be oriented in the laboratory, symbolically indicated by the black arrow, to probe specific "preferred" directions as discussed with reference to fig. 3.

may choose to fit the lifetimes of the sample instead, as noted in the last equality. We assume that the electromagnetic and strong interaction are not breaking Lorentz symmetry. The yield of photons may depend on the degree of polarization but not on its sign. Detecting γ's has, therefore, the advantage that systematic errors due to parity violation are eliminated, while evaluating eq. (4) from the β-particle yield might lead to errors mimicking LIV.

A generic setup is shown in fig. 2 which measures simultaneously the asymmetry in β and γ emission from a sample where the polarization can be efficiently reversed. Many of the systematic errors can be eliminated with high precision in this highly symmetric setup cf. [16]. Although there are many isotopes that can be studied with the strategy described above, the requirement of polarization and the demand of a high source strength restricts the isotope choice.

The direction of polarization, $\vec{j}$, should be chosen in the context of systematic errors.

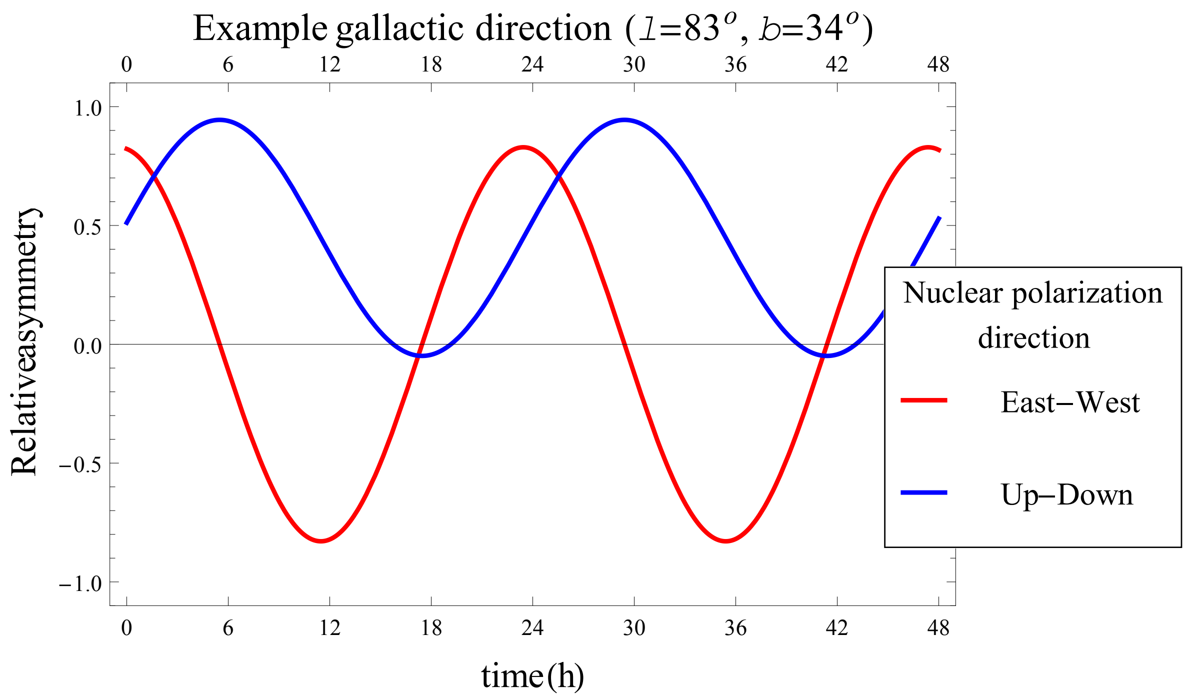

FIG. 3: Example of a lifetime asymmetry as function of time for two polarization directions with an arbitrary chosen "preferred direction" as indicated

For example, choosing the polarization parallel to the earth's rotation axis would result in an asymmetry of the γ yield independent of the time of day. In contrast, orienting $\vec{j}$ in the east-west direction means the asymmetry would show as a sinusoidal dependence around zero with the pattern reversing sign every half rotation of the earth. Two cases are shown in fig. 3 for a hypothetical "preferred direction"; these are polarizations in the east and west directions and polarization in the up and down directions, i.e. perpendicular to the earth's surface. In the latter case a signal would have a zero offset and the sinusoidal dependence would be shifted with respect to the east-west case. It appears that the east-west configuration has to be preferred with respect to systematic errors, because a constant offset is difficult to discriminate from a systematic bias. However, in this east-west configuration, no signal would be observed if $\hat{n}_2$ is parallel to the earth's rotation axis. Therefore, more than one orientation of the setup of fig. 2 should be considered.

Measuring ξ_2 requires a highly active sample that can be polarized. Our laboratory has an intense solid-state laser for efficient trapping of radioactive ^{21}Na in a magneto-optical trap [17]. This laser can also be used for polarizing large samples of any Na isotope. Polarizing

the sample in a buffer gas avoids the loss mechanisms associated with capturing ions, and neutralizing them for atomic trapping, hence the buffer-gas method is preferred. The decay scheme shown in fig. 1 is available in $^{20,24-27}$Na. Of these ^{20,26,27}Na are useful in our setup (see below), but only ^{20}Na can be produced in excess of 10^6 particles per second at our facility, for this reason ^{20}Na was selected for this study.

In this experiment we look for a change in the decay rate of the allowed β decay of ^{20}Na when reversing the orientation of the nuclear spin $\vec{J}$ via optical pumping. ^{20}Na is produced via the ^{20}Ne(p,n)^{20}Na reaction by colliding a ^{20}Ne beam with hydrogen in a gas target [18]. The resulting isotopes pass through the TRIμP isotope separator facility to obtain a ^{20}Na beam which is stopped in a buffer gas cell filled with up to 8 atm of neon gas.

Adjustable aluminum degrader foils in the beam line allow to position the beam's stopping distribution in the center of the gas cell. The neon buffer gas is cleaned with a cryo-trap filled with liquid nitrogen and a gas purifier cartridge. A heatable dispenser with natural sodium is mounted inside the buffer gas cell. The use of the dispenser proved to be essential. The natural sodium binds residual chemically active contaminants in the gas that would bind radioactive sodium into molecules, making them unavailable for polarization.

The stopped ^{20}Na atoms in the center of the buffer gas cell are optically pumped into a "stretched" state in which the electronic and nuclear spins are both aligned along the direction of the magnetic holding field provided by Helmholtz coils. To achieve this, a circularly polarized laser beam with 589 nm wavelength is sent through the buffer gas cell. Remotely operated beam blockers allow to switch the polarization of the laser light going to the cell. Depending on the helicity of the light that enters the buffer gas cell, the atoms will be pumped into a state with the spins aligned or anti-aligned to the direction of the magnetic field. The polarization is measured from β^+ rates as shown in fig. 4 using eq. (3). For better control of systematic errors we operate the setup in three short cycles of 4 seconds, one for each spin polarization direction and one with the laser beam off. Within each cycle the beam is on and off for two seconds. A maximum polarization of about 50% was obtained. The polarization decreases when the beam is off, presumably due to the drift of particles out of the volume covered by the laser. For this reason a short lifetime is advantageous; The halflife of ^{20}Na is 448 ms. These and other factors playing a role in the experimental method will not be discussed further here but in a forthcoming publication [13].

Fig. 4 shows also the measured γ rates. When the sample is polarized, a small enhancement of the γ emission can be observed, which is independent of the sign of the polarization.

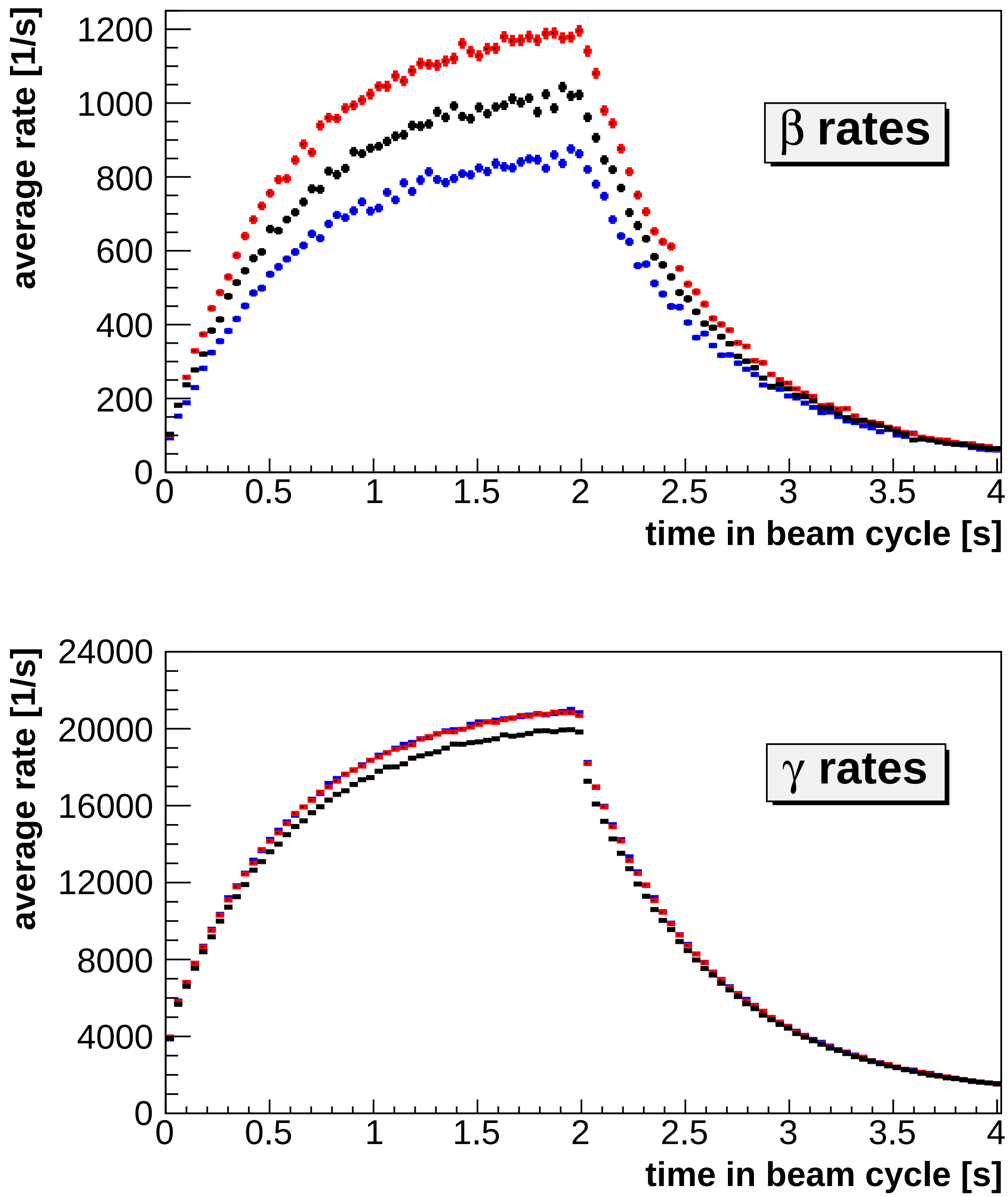

FIG. 4: β (top) and γ (bottom) rates measured with laser polarization in two directions (indicated by red and blue data points, the latter mostly covered by the former) and without laser (black data points). The periods with beam on and off are 2 seconds.

This is due to the quadrupole emission pattern of the γ-rays following the β decay of the polarized parent nucleus. Of course this signal may not depend on the sign of the polarization. A first set of data for up-down polarization has been analyzed and the results will be available soon [13]. In this data set the precision is at a level of 10^{-3}.

In the following we give some details of the theoretical work that we use to give more physical meaning to the experimental observables. Indeed, to have limits on $\xi_i \hat{n}_i$ without being able to relate them to each other or to some underlying theory is not satisfactory. For this reason a version of the SME was formulated applicable for weak decays. We use an extension where only the W propagator is modified. Such a specific choice, of course, can not cover all theoretical possibilities, but can be quite well motivated and is most appropriate for β decay. This theoretical work is discussed in ref. [8]. Here, we restrict the discussion to the results of this theoretical exploration, in particular for the parameters $\xi_i \hat{n}_i$.

The Lorentz-violating propagator at low energies that we use is given by

$$\left\langle W^{\mu+}(p) W^{\nu-}(-p) \right\rangle = \frac{-i(g^{\mu\nu} + \chi^{\mu\nu})}{M_W^2} \ , \tag{5}$$

where $g^{\mu\nu}$ is the Minkowski metric and $\chi^{\mu\nu}$ is a general Lorentz-violating (complex, possibly momentum-dependent) tensor. Neglecting the dependence on boosts, i.e. assuming that the velocities of the parent nuclei are small with respect to the "preferred frame", we find that the parameter for the anisotropy of the emission direction is given by

$$\xi_1 \hat{n}_1^l = 2\chi_r^{0l} \tag{6}$$

for Fermi transitions, while for Gamow-Teller transitions

$$\xi_1 \hat{n}_1^l = \frac{2}{3}(\chi_r^{l0} + \epsilon^{lmk}\chi_i^{mk}). \tag{7}$$

Here the subscript r (i) refers to the real(imaginary) part of χ; the notation is such that, for example, the right-hand side of eq. 7 has the x component $\frac{2}{3}(\chi_r^{10} + \chi_i^{23} - \chi_i^{32})$.

Thus, a striking result of this theory is that there is no single "preferred direction". It differs for Fermi and Gamow-Teller transitions, but it is also different for other observables. How $\xi_1 \hat{n}_1$ depends on $\chi^{\mu\nu}$ for forbidden β decays, is presently evaluated. An enhancement for a nucleus with atomic number Z and radius R can be expected of order $\alpha Z/R \approx 0.3Z$, i.e. an order of magnitude larger than the LIV effect in allowed decays [12]. The underlying idea of ref. [10, 11] that angular momentum may not be conserved in the weak interaction can thus

be made quantitative in the present theory. The theory also shows that searching for LIV in second-forbidden reactions as in [11] gives no further enhancement over first-forbidden decays.

Gamow-Teller transitions allow one to explore most of the parameter space contained in χ, in particular when measuring β-spin correlations as function of direction. The full expression is given in ref.[8]. The parameter ξ_2, which gives the dependence on spin orientation, is relatively simple:

$$\xi_2 \hat{n}_2^l = A \epsilon^{lmk} \chi_i^{mk}, \tag{8}$$

where A is the SM β-asymmetry parameter. In terms of the generic measurement of ξ_2 and thus also for the experiment on ^{20}Na described here, one finds that

$$j^l \epsilon^{lmk} \chi_i^{mk} = \frac{R_\beta^\uparrow + R_\beta^\downarrow}{R_\beta^\uparrow - R_\beta^\downarrow} \frac{R_\gamma^\uparrow - R_\gamma^\downarrow}{R_\gamma^\uparrow + R_\gamma^\downarrow}. \tag{9}$$

This may suggest that the experimental method just consists of measuring the rates in the detectors shown in fig. 2. However, in particular for a short-lived sample, one would need to guarantee that conditions during the two polarization periods are identical.

For completeness we note that in our theory, for any allowed transition,

$$\xi_3 \hat{n}_3 = \mp \sqrt{(1-(\alpha Z)^2)(1-\beta^2)} \xi_1 \hat{n}_1, \tag{10}$$

where $\mp$ refers to the case of $\beta^\mp$ decay. Measuring the β polarization is not well possible with high efficiency. Moreover, $\xi_3 \hat{n}_3 < \xi_1 \hat{n}_1$. In this respect nothing is gained over measuring the β-emission direction only.

Of course our theory need not be restricted to β decay but can be evaluated for any weak interaction involving the W boson. In this respect it is interesting to note that the KLOE collaboration has measured the lifetime of K_S mesons as function of the kaon emission direction with respect to the dipole anisotropy of the Cosmic Microwave Background [21]. We find [22] that their search for anisotropy is complementary to β decay, however, to gain the maximal information on $\chi^{\mu\nu}$ a reanalysis of their data would be required.

In summary, we have identified, in the context of experimental searches for physics beyond the Standard Model, an important class of tests for Lorentz symmetry breaking in the weak interaction. These tests can be made in experiments exploiting the properties of β decay. First experiments have been done. A theoretical framework that considers modification of

the W propagator allows one to put the various tests in context and relate them to each other and to non-leptonic weak decays. Part of this theoretical program has been completed. The theory has a rich structure, for example that there need not be a single preferred direction for Lorentz invariance violation. A variety of experiments is required to limit its parameters $\chi^{\mu\nu}$ that, in turn, can be related to parameters of the SME model.

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